

Brazilian Sophie Germain Primes

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Present Your Science Coaching Session

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I'm going to lead you through the origins of a conjecture and how we used mathematics and computing to solve it.

 +  = ? (Definitions)

$$2 \times (2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 1) + 1, \text{ etc. (A Conjecture)}$$

$$28792661 = 73^4 + 73^3 + 73^2 + 73 + 1 \text{ (Math \& Computation)}$$

 and  (Bateman-Horn Conjecture)

Sophie Germain primes are primes p where $2p + 1$ is also prime.



Brazilian primes are primes that are all 1s in some base b .



Example: 1111111111111111111.

Smaller example: $7 = 2^2 + 2 + 1$.

Non-example: 11. (Length must be ≥ 3 .)

Non-example: $3 = 1^2 + 1 + 1$. (b can't be 1)

Bernard Schott (2010) conjectured that there are no Brazilian Sophie Germain primes.

$$7 = 2^2 + 2 + 1$$

$$2 \times 7 + 1 = 3 \times 5$$

$$13 = 3^2 + 3 + 1$$

$$2 \times 13 + 1 = 3 \times 9$$

$$31 = 5^2 + 5 + 1$$

$$2 \times 31 + 1 = 3 \times 21$$

$$43 = 6^2 + 6 + 1$$

$$2 \times 43 + 1 = 3 \times 29$$

$$73 = 8^2 + 8 + 1$$

$$2 \times 73 + 1 = 3 \times 49$$

$$127 = 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 1 \quad 2 \times 127 + 1 = 3 \times 85$$

$$157 = 12^2 + 12 + 1$$

$$2 \times 157 + 1 = 3 \times 105$$

$$211 = 14^2 + 14 + 1$$

$$2 \times 211 + 1 = 3 \times 141$$

$$241 = 15^2 + 15 + 1$$

$$2 \times 241 + 1 = 3 \times 161$$

Certain lengths, like 3 and 7, can be excluded by mathematical proof, so we looked at length 5.

$$28792661 = 73^4 + 73^3 + 73^2 + 73 + 1$$

$$78914411 = 84^4 + 84^3 + 84^2 + 84 + 1$$

...

104,890,280 length-5 examples up to 10^{46} .

$$\begin{aligned} 14781835607449391161742645225951 = \\ 1309^{10} + 1309^9 + 1309^8 + 1309^7 + 1309^6 + 1309^5 + \\ 1309^4 + 1309^3 + 1309^2 + 1309 + 1. \end{aligned}$$

22 length-11 examples up to 10^{46} .

The Bateman–Horn Conjecture predicts the behavior of primes of special form.



$$\infty$$

$$\sim C \frac{x^{1/4}}{\log x^2}$$

Interesting mathematical and computational techniques
enable progress on an elementary problem.

0 1 3 6 2 7
: 13
: 20
23 12
10 22 11 21
THE ON-LINE ENCYCLOPEDIA
OF INTEGER SEQUENCES[®]
founded in 1964 by N. J. A. Sloane

[Hints](#)
(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

A306845 Sophie Germain primes which are Brazilian. ³
28792661, 78914411, 943280801, 7294932341, 30601685951, 919548423641, 2275869057821,
4172851565741, 4801096143881, 27947620155401, 29586967653101, 43573806645461,
119637719305001, 124484682222941, 148908227169101, 172723673300501 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#);
[internal format](#))
OFFSET 1,1
COMMENTS These terms point out that the conjecture proposed in Quadrature "No Sophie
Germain prime is Brazilian (prime)" (see link) was false.

Fun Background



Mathematical Insights



Computational Techniques



Publishable Research